Holographic hydrodynamics at finite coupling

Aninda Sinha, Perimeter Institute, Canada.

based on 1) arXiv:0806.2156 with Rob Myers and Miguel Paulos

2) arXiv:0808.1837 with Alex Buchel, Rob Myers and Miguel Paulos

3) arXiv:0809.xxxx with Alex Buchel and Rob Myers.

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- Review of η/s

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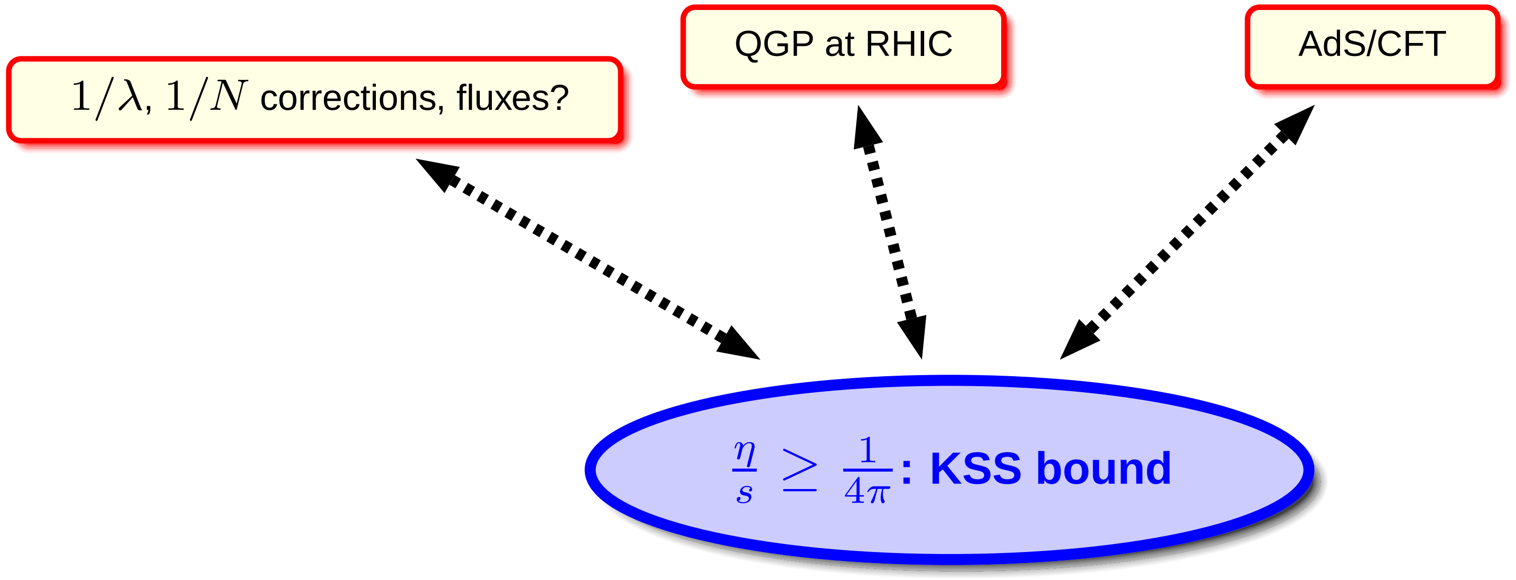
- Discussion

$\frac{\eta}{s}$ is interesting



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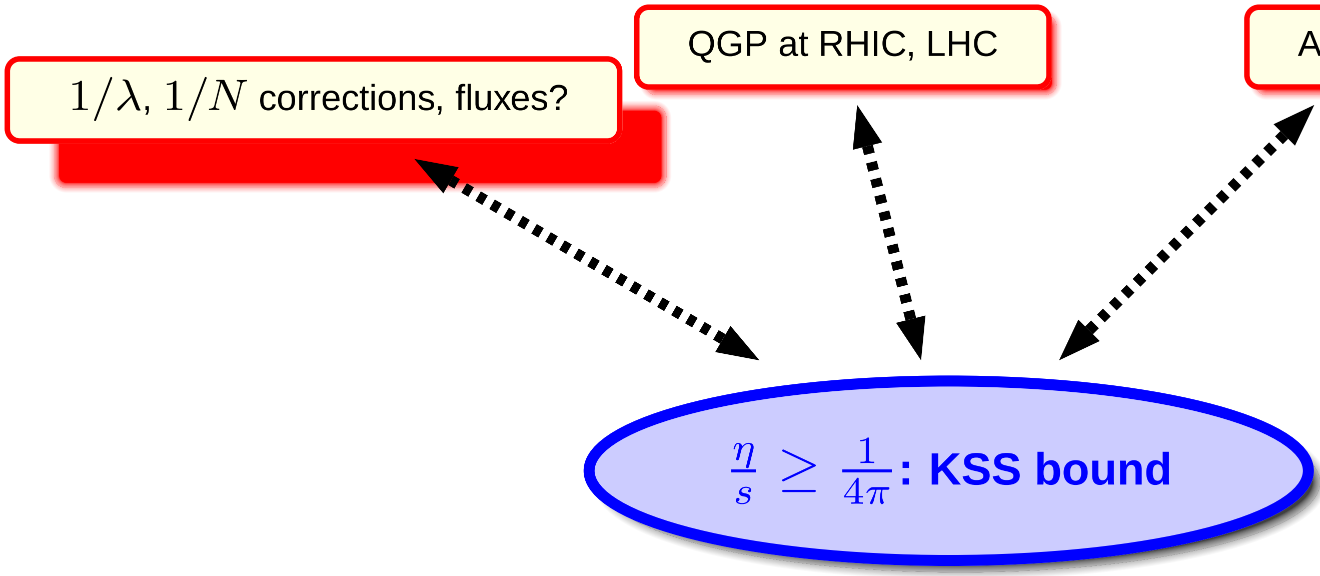

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- For trapped strongly-interacting Fermi gas of Lithium-6 atoms, $\frac{\eta}{s} > 0.4$ **[Schafer, 2007]**.

REVIEW OF $\frac{\eta}{s}$

- For $\mathcal{N} = 4$ super-Yang Mills start with gravity dual $AdS_{BH} \times S^5$

$$\begin{aligned} ds^2 &= \frac{(\pi T L)^2}{u}(-f dt^2 + d\mathbf{x}^2) + \frac{L^2 du^2}{4u^2 f} + L^2 dS^2_{\mathcal{M}_5} \\ F_5 &= -\frac{4}{L}(1+\star)\text{vol}_{\mathcal{M}_5}, \qquad \mathbf{x} = (x, y, z)\,. \end{aligned} \tag{1}$$

Here $f = 1 - u^2$. Horizon $u = 1$, boundary $u = 0$.

- Add perturbation $h_{xy} = \phi(u) e^{-i\omega t}$. Solve resulting differential equation with incoming boundary conditions at the horizon ($\omega/T \ll 1$).

- Use Kubo formula

$$\eta = -\lim_{\omega \to 0} \frac{1}{\omega} Im\, G^R_{xy,xy}(\omega, \mathbf{0})\,. \tag{2}$$

- Get

$$\boxed{\frac{\eta}{s} = \frac{1}{4\pi}}$$

QUANTUM CORRECTIONS

- Quantum corrections to this formula are obtained by considering higher derivative α' and string loop g_s corrections.

- Buchel, Liu, Starinets consider the well known C^4 term

$$S_{R^4}^{(3)} = \frac{\gamma}{16\pi G}\int d^{10}x\,\sqrt{-g}\,e^{-\frac{3}{2}\tilde{\phi}}\,W_{C^4} \tag{3}$$

$$W_{C^4} = \underbrace{C_{abcd}}_{\text{Weyl tensor}} C^{ebcf}C^{agh}{}_{e}C^{d}{}_{ghf} - \frac{1}{4}C_{abcd}C^{ab}{}_{ef}C^{ce}{}_{gh}C^{dfgh}$$

where $\gamma = \frac{1}{8}\zeta(3)\alpha'^3$. They obtain

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 + \frac{15\zeta(3)}{\lambda^{3/2}}\right). \tag{4}$$

- However, by dimensional analysis there are many, many other terms at the same order α'^3. What role do these play?

- Also, can we say something about $1/N$ corrections in addition to $1/\lambda$ corrections?

Flux terms at α'^3

- So what are known about other terms? Green and Stahn postulated in 2003 that if only the metric and 5-form are involved then all the terms at α'^3 can be packaged into a neat form

$$S^{(3)}_{\mathcal{R}^4} = \frac{\alpha'^3 g_s^{3/2}}{32\pi G}\int d^{10}x \int d^{16}\theta \sqrt{-g} f^{(0,0)}(\tau,\bar\tau)[(\theta\Gamma^{mnp}\theta)(\theta\Gamma^{qrs}\theta)\mathcal{R}_{mnpqrs}]^4 + c.c. \quad (5)$$

where $f^{(0,0)}(\tau,\bar\tau)$ is a modular form [Green,Gutperle]and the six-index tensor $\mathcal{R}$ is specified by

$$\mathcal{R}_{mnpqrs} = \frac{1}{8} g_{ps} C_{mnqr} + \frac{i}{48}\nabla_m F^+_{npqrs} + \frac{1}{384} F^+_{mnptu} F^{+\ tu}_{qrs}\,, \quad (6)$$

with $F^+ = \frac{1}{2}(1+*)F_5$.

- If other fields are turned on at leading order the above is no longer true since there exists no chiral measure in superspace [Berkovits, Howe, de Haro, Sinkovics, Skenderis].
- If one considers this full set of terms, there are no corrections to the D3-brane solution [Green, Stahn]but if one neglects them, there are corrections [de Haro, Sinkovics, Skenderis].
- So it seems to be crucial not to ignore them!

• Integrating over superspace a very hard job. Need some nice computer package. Fortunately there is one by Kasper Peeters called Cadabra. Using this Miguel Paulos showed that the superspace integral results in 20 independent terms which can be written as

$$C^4 + C^3\mathcal{T} + C^2\mathcal{T}^2 + C\mathcal{T}^3 + \mathcal{T}^4 \;+\; c.c.\,, \tag{7}$$

with

$$\mathcal{T}_{abcdef} \equiv i\nabla_a F^+_{bcdef} + \frac{1}{16}\left(F^+_{abcmn}F^{+\;mn}_{def} - 3F^+_{abfmn}F^{+\;mn}_{dec}\right). \tag{8}$$

Antisymmetry in $[a, b, c], [d, e, f]$ and symmetry under exchange of triples is implied.

• We will add a deformation h_{xy}, solve the lowest order eom and use this in the α'^3 terms. A huge simplification occurs since

$$\mathcal{T} = 0\,. \tag{9}$$

even in the deformed background!

• So only need to consider

$$W_{C^3\mathcal{T}} = \frac{3}{2} C_{abcd}\, C^a{}_{efg}\, C^{bf}{}_{hi}\, \mathcal{T}^{cdeghi} \;+\; c.c.\,. \tag{10}$$

The following observation simplifies life a lot. Firstly the lowest order 10-d equations of motion read

$$R_{ab} = \frac{1}{96} F_{a d_1 d_2 d_3 d_4} F_b{}^{d_1 d_2 d_3 d_4} \,. \tag{11}$$

Further self-duality of F implies $R = 0$ and since F is proportional to the volume-form

$$R_{ab} \propto g_{ab} \tag{12}$$

i.e., the deformed manifolds are Einstein to lowest order with *equal and opposite* curvatures. Note that to make this observation we don't really care about the actual form of the metric but only that it is of the type

$$A_5 \times \mathcal{M}_5$$

- Now since

$$C_{abcd} = R_{abcd} - \underbrace{\frac{1}{4}(g_{a[c}R_{d]b} - g_{b[c}R_{d]a})}_{=0 \text{ if indices not all } A_5 \text{ or } \mathcal{M}_5} + \frac{1}{36}\underbrace{R}_{=0}\, g_{a[c}g_{d]b} \tag{13}$$

and R_{abcd} has components only along the 5d black hole direction or the 5d compact space direction

$$C \to C_{a_1 a_2 a_3 a_4} \quad \text{or} \quad C_{s_1 s_2 s_3 s_4}$$

• This means that all the 6 indices lie either in A_5 or in $\mathcal{M}_5$. But 3 indices of this contracts into F_5 which also has all indices either in A_5 or in $\mathcal{M}_5$. This means that the only possible index structure for C^3 is

$$(C^3)_{a_1 a_2 a_3 a_1 a_2 a_3}$$

• Now in $C^3\mathcal{T}$,

$$\left((C^3)_{abc}{}^{def} - 3(C^3)_{[ab}{}^{[fde]}{}_{c]} \right) .$$

and this VANISHES for the above index structure!!!

- Given that $P(C^3)F_5 = 0 = \mathcal{T}$, it also follows $\delta W_{C^3\mathcal{T}}/\delta g_{ab}$ vanishes.

We conclude that only C^4 alters the geometry at order α'^3.

$\frac{\eta}{s}$ is only corrected by C^4 at this order.

POINTS TO REMEMBER

- The argument does not care about what $\mathcal{M}_5$ actually is so long as it is Einstein with equal and opposite curvature to A_5 at leading order. So it applies to L_{pqr} manifolds.

- The argument is to all orders in the deformation.

- It is thus sufficient to only include the C^4 term in calculating various transport coefficients when discussing the AdS-Schwarzschild $\times L_{pqr}$ manifold.

- Thus one can probe questions like universality (or does $\frac{\eta}{s}$ care about what p, q, r actually are at this order?)

- This argument relied on the product form $A_5 \times \mathcal{M}_5$ and hence does not apply to R-charged black holes.

One very important type of corrections which are necessary to compute to be able to say anything useful (if at all!) about real world QCD are $1/N$ corrections.

It turns out that there is a straightforward way in which one can compute the leading $1/N$ correction.

For this note that SL(2,Z) symmetry fixes the coefficient of C^4 term to be a modular form. Remarkably [Green, Gutperle] the g_s perturbative piece comprises only of 2 terms:

$$f_P^{(0,0)} = \frac{\zeta(3)}{8} e^{-3\phi/2} \left(1 + \frac{\pi^2}{3\zeta(3)} e^{2\phi}\right) . \tag{14}$$

• Since the dilaton is only sourced at α'^3 we can simply extend the existing result to include this term. This gives

$$\frac{\eta}{s} = \frac{1}{4\pi} \left(1 + \frac{15\zeta(3)}{\lambda^{3/2}} + \frac{5}{16} \frac{\lambda^{1/2}}{N^2} + \tilde{f}_{NP}\right)$$

Using $\lambda = 6\pi, N = 3$ this gives $\frac{\eta}{s} = 0.11$ (changed from 0.08).

Buchel and Liu earlier argued that the ratio is universal for a large class of gauge theories with gravity duals. The question we would like to ask is what is the finite coupling correction if we consider L_{pqr}. A bit surprisingly the answer turns out to be universal in terms of the gravity variables α', g_s.

STRATEGY:

- Reduce 10-d C^4 down to 5-dimensions.
- Resulting action works out to be completely independent of the compact manifold.

Start with formula for d-dimensional Weyl tensor:

$$C_{abcd} = R_{abcd} - \frac{2}{d-2}\left(g_{a[c}\,R_{d]b} - g_{b[c}\,R_{d]a}\right) + \frac{2}{(d-2)(d-1)}R\,g_{a[c}\,g_{d]b}\,. \quad (15)$$

In the present background with the product form, $A_5 \times \mathcal{M}_5$, we have

$$\begin{aligned} \tilde{C}_{abcd} &= C_{abcd} + 10\left(g_{a[c}Y_{d]b} - g_{b[c}Y_{d]a}\right) + 2X g_{a[c}g_{d]b} \\ \tilde{C}_{mnpq} &= \hat{C}_{mnpq} + 2X \hat{g}_{m[p}\hat{g}_{q]n} \\ \tilde{C}_{manb} &= -3Y_{ab}\,\hat{g}_{mn} - \frac{4}{5}X\, g_{ab}\,\hat{g}_{mn} \end{aligned} \qquad (16)$$

where we have defined

$$\begin{aligned} Y_{ab} &\equiv \frac{1}{24}\left(R_{ab} - \frac{1}{5}R\, g_{ab}\right) \\ X &\equiv \frac{1}{72}(R + \widehat{R})\,. \end{aligned} \qquad (17)$$

It will be important in what follows that Y and X vanish when evaluated on the leading order supergravity solution and also that Y is traceless (in general), i.e., $Y^a{}_a = 0$.

Schematically we get

$$\tilde{C}^4 = C^4 + \hat{C}^4 + \hat{C}^3 X + C^3 Y + C^3 X + \mathcal{O}(Y^2, X^2, XY)\,, \qquad (18)$$

Since $X = Y = 0$ on-shell, we only need to worry about the linear terms in X, Y. The $\hat{C}^3 X$ term has the explicit form:

$$4X\left(2\,\hat{C}_{mnpq}\,\hat{C}^{mp}{}_{rs}\,\hat{C}^{nrqs} - \hat{C}_{mnpq}\,\hat{C}^{m}{}_{r}{}^{p}{}_{s}\,\hat{C}^{nrqs}\right)\,. \tag{19}$$

This works out to be zero–these kinds of identities are called SCHOUTENidentities and can be proved by antisymmetrizing $d+1$ indices in d dimensions.

Hence, the compact manifold plays no role in the equations of motion. Thus all hydrodynamic quantities will be the same even at finite coupling in terms of the string variables.

Trace anomaly coefficients for superconformal gauge theories are of 2 types, a and c. The conformal anomaly of a four-dimensional QFT takes form

$$\langle T^\mu_\mu \rangle_{CFT} = \frac{c}{16\pi^2} I_4 - \frac{a}{16\pi^2} E_4 \,, \tag{20}$$

where c and a are the CFT central charges and

$$E_4 = R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda} - 4R_{\mu\nu}R^{\mu\nu} + R^2 \,, I_4 = R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda} - 2R_{\mu\nu}R^{\mu\nu} + \frac{1}{3}R^2 \,, \tag{21}$$

Until now all results were for theories with $a = c$ or with adjoint matter only. What about $a \neq c$?

<u>STRATEGY</u>: Use an effective action approach. R^2 terms arise as α' corrections to DBI action [Bachas,Green,Bain]. Consider an action

$$S = \int d^5x \sqrt{-g}\left(\frac{1}{\kappa^2} R - \Lambda + c_1 R_{abcd}R^{abcd} + c_2 R_{ab}R^{ab} + c_3 R^2 + \cdots\right) , \tag{22}$$

It was shown by [Kats and Petrov, Brigante et al]that

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 - \frac{8c_1\kappa^2}{\ell^2} + \cdots\right) , \tag{23}$$

SO IF $c_1 > 0$, viscosity bound is violated. It turns out that holographic Weyl anomaly calculations [Henningson, Skenderis, Nojiri, Odinstov] relates $c_1 \propto c - a$. So the question can be turned around:

ARE THERE ANY EXAMPLES WHERE $c - a > 0$ IN A CONTROLLABLE SETTING?

The answer is surprisingly: IN ALL CASES THAT WE KNOW!!

EXAMPLES:

- SU(N) with matter content in $\mathcal{N} = 1$ susy language $(n_{adj}, n_{asym}, n_{sym}, n_f) = (2,1,0,1),(1,2,0,2),(1,1,1,0),(0,3,0,3),(0,2,1,1)$.
- Sp(2N) with matter content in $\mathcal{N} = 1$ susy language $(n_{adj}, n_{asym}, n_f) = (2,1,4),(1,2,8),(0,3,12)$

Also certain examples in $\mathcal{N} = 2$ SCFTs with isolated superconformal fixed points (D7 branes at F-theory singularities) studied by [Aharony, Tachikawa].

DISCUSSION

● Can use only the C^4 term for a variety of calculations of transport coefficients in SYM. Can also use SL(2,Z) to get quantum corrections in $\sqrt{\lambda}/N^2$.

● Hydrodynamics seems to be universal at finite coupling for a large class of theories.

● Adding fundamental matter seems to lead to a $O(1/N)$ violation–probably concluding that the bound is violated for real world extrapolations is premature since there $O(1/\lambda^{3/2}) \sim O(1/N)$.

● Is there a new bound [Brigante, Liu, Myers, Shenker, Yaida]

$$\frac{\eta}{s} \geq \frac{16}{25}\frac{1}{4\pi}????$$

OR is there no bound

$$\frac{\eta}{s} \geq 0?????$$

Please place your bets with Rob or me. THANK YOU FOR YOUR ATTENTION.